ACTION OF A MOVING LOAD ON A NONLINEARLY COMPRESSED PLASTIC STRIP WITH A DEFORMABLE BASE

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The two-dimensional problem of the action of a moving load on a nonlinearly-compressible plastic strip lying in a half-space of linearly elastic or plastic material is considered. Rocks and protective liners of various underground structures may be considered as the base of the strip, in particular a layer of soft ground.

It is noted that the problem of action of a moving load on a linearly compressible plastic strip with a rigid base was examined in [1, 2]. In contrast to [1, 2] in this work the wave process in a layered material is studied taking account of nonlinear loading of the strip material and the stress—strained state of the base, the effect of inelastic properties of the materials on the distribution of kinematic parameters and stresses in them is studied, and the shape of the front surface of the wave reflected from an elastic rocky base is determined.

1. Let over the upper boundary of a strip of thickness h move a steadily decreasing normal load with constant velocity D exceeding the velocity of loading—unloading deformation propagation for the medium and the base. The medium filling strip is modelled by a generalized "plastic gas" [3, 4] and under load the connection between pressure p and volumetric deformation ε is taken in the form $p = \alpha_1\varepsilon + \alpha_2\varepsilon^2$ ($dp/d\varepsilon > 0$, $d^2p/d\varepsilon^2 > 0$). The angle of slope of the unloading branch E for the $p \sim \varepsilon$ diagram exceeds the slope of the loading branch, and the loading profile does not change as the wave propagates.

If the material of the base of the strip is linearly elastic and the medium is dense, i.e., $\rho_0 < \rho_{ba}$ (ρ_0 and ρ_{ba} are density of the strip and base materials), then the compression wave with a curvilinear surface propagating in the strip Σ (Fig. 1) with $\xi = x + Dt \geq \xi_a$, $\eta = y = h$ after reaction with the base generates in it elastic longitudinal and transverse waves and also a reflected shock wave from the surface Σ_0 in the strip ahead of which with considerable velocity $c_p = \sqrt{E/\rho_0}$ a weak distortional elastic wave radiates as a characteristic of the negative direction. As a result of wave propagation and reaction with the strip boundaries disturbed areas 1-4, and I, II (Fig. 1) arise respectively. Solution of the problem for regions 1 and 2 by an inverse method in the case when the prescribed shape of the front surface Σ has the form $\eta(\xi) = (R_1 - R_2\xi/2)\xi$ (R_1, R_2 are prescribed constant values) has been obtained previously in [2]. Given below is an analytical solution of the contact problem for regions 3, and I, II.

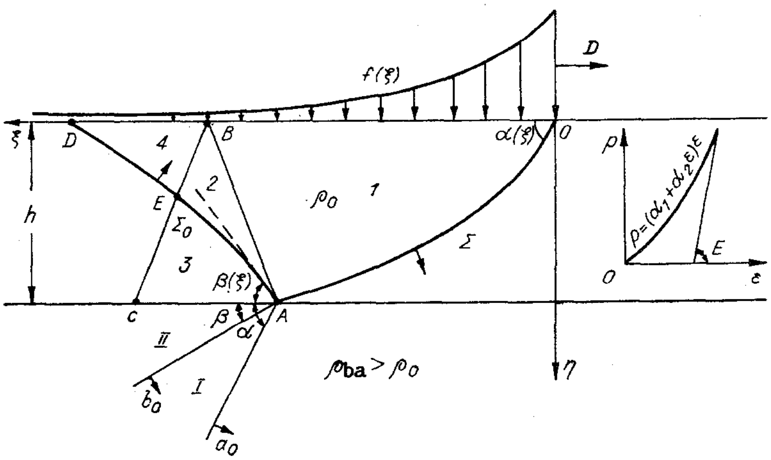

Fig. 1

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Solution of the problem in region 3 in relation to velocity potential $\varphi_3(\xi, \eta)$, as in [2], is expressed by a d'Alembert equation

$$\varphi_3(\xi, \eta) = f_3(\xi - \mu\eta) + f_4(\xi + \mu\eta), \tag{1.1}$$

$u_3(\xi, \eta) = \partial\varphi_3/\partial\xi$, $v_3(\xi, \eta) = \partial\varphi_3/\partial\eta$ are horizontal and vertical ground velocity components in region 3.

In elastic regions I and II of a rocky half-space for potentials of displacements Φ and ψ according to [5] with $D > a_0$

$$\Phi(\xi, \eta) = F_3(\xi - \mu_1\eta),\ \psi(\xi, \eta) = F_4(\xi - \mu_2\eta), \tag{1.2}$$

where $F_3(0) = F_4(0) = 0$; $\mu_1{}^2 = (D/a_0)^2 - 1$; $\mu_2{}^2 = (D/b_0)^2 - 1$; $a_0{}^2 = (\lambda + 2G)/\rho_{ba}$; λ and G are Lamé coefficients for the base material. In order to find unknown functions $f_3(z)$, $f_4(z)$, $F_3(z)$ and $F_4(z)$ the problem in regions 3, I, and II has the following boundary conditions: at the reflected wave front

$$\rho_2^*(a_{re} - v_{2n}^*) = \rho_3^*(a_{re} - v_{3n}^*),\ \rho_2^*(a_{re} - v_{2n}^*)(v_{2n}^* - v_{3n}^*) = p_2^* - p_3^*,$$
$$v_{2\tau}^* = v_{3\tau}^*,\ a_{re} = D\sin\beta(\xi),\ p_j^* = \alpha_1\varepsilon_j^* + \alpha_2\varepsilon_j^{*2},\ \varepsilon_j^* = 1 - \rho_0/\rho_j^*,\ j = 2, 3; \tag{1.3}$$

at the contact of the two media with $\eta = h$, $\xi_a \le \xi \le \xi_0$

$$\sigma_{\xi\eta} = 0,\ \sigma_{\eta\eta} = -p_3(\xi, \eta),\ D\left(\frac{\partial^2\Phi}{\partial\xi\,\partial\eta} + \frac{\partial^2\psi}{\partial\xi^2}\right) = \frac{\partial\varphi_3}{\partial\eta}, \tag{1.4}$$

where

$$U = \frac{\partial\Phi}{\partial\xi} - \frac{\partial\psi}{\partial\eta};\ V = \frac{\partial\Phi}{\partial\eta} + \frac{\partial\psi}{\partial\xi};\ \varepsilon_{\xi\xi} = \frac{\partial U}{\partial\xi};\ \varepsilon_{\eta\eta} = \frac{\partial V}{\partial\eta};\ \varepsilon_{\xi\eta} = \frac{\partial U}{\partial\eta} +$$
$$+ \frac{\partial V}{\partial\xi};\ \varepsilon = \varepsilon_{\xi\xi} + \varepsilon_{\eta\eta};\ \sigma_{\eta\eta} = \lambda\left(\frac{\partial^2\Phi}{\partial\xi^2} + \frac{\partial^2\Phi}{\partial\eta^2}\right) + 2G\left(\frac{\partial^2\Phi}{\partial\eta^2} + \frac{\partial^2\psi}{\partial\xi\,\partial\eta}\right);$$
$$\sigma_{\xi\eta} = G\left(2\frac{\partial^2\Phi}{\partial\xi\,\partial\eta} + \frac{\partial^2\psi}{\partial\xi^2} - \frac{\partial^2\psi}{\partial\eta^2}\right);\ v_n = -\frac{\partial\varphi}{\partial\xi}\sin\beta -$$
$$- \frac{\partial\varphi}{\partial\eta}\cos\beta;\ v_\tau = -\frac{\partial\varphi}{\partial\xi}\cos\beta + \frac{\partial\varphi}{\partial\eta}\sin\beta. \tag{1.5}$$

In Eqs. (1.3)-(1.5) there is additional use of the following notations: U and V are horizontal and vertical components of elastic medium displacement; $\sigma_{\eta\eta}$ and $\sigma_{\xi\eta}$ are components of elastic stresses; v_n and v_τ are normal and tangential components of mass velocity for the strip material in relation to front Σ_0; $\beta(\xi)$ is angle of slope for the reflected wave front with axis $O\xi$ ($\beta_0 \approx \beta(0)$), which is subject to determination in the course of solving the problem; a_{re} is reflected wave front propagation velocity; parameters relating to front Σ_0 are marked with an asterisk.

Assuming in the first approximation $\beta(\xi) \approx \beta_0$, the third equation of (1.3) with $n \approx h - \tan\beta_0\ (\xi - \xi_a)$ is written in the form

$$\left(\frac{\partial\varphi_2}{\partial\xi} - \frac{\partial\varphi_3}{\partial\xi}\right) = \operatorname{tg}\beta_0\left(\frac{\partial\varphi_2}{\partial\eta} - \frac{\partial\varphi_3}{\partial\eta}\right). \tag{1.6}$$

If it is considered that $a_{re} \gg v_{2n}{}^*$ and $\rho_2{}^* \approx \rho_0$, then the first and second equations of (1.3) are written as

$$-\rho_0 D(\partial\varphi_2/\partial\xi - \partial\varphi_3/\partial\xi) = p_2{}^* - p_3{}^*; \tag{1.7}$$

$$\operatorname{tg}\beta(\xi) = \frac{\left[\left(\frac{\partial\varphi_2}{\partial\eta} - \frac{\partial\varphi_3}{\partial\eta}\right) + \varepsilon_2^*\frac{\partial\varphi_3}{\partial\eta} - \varepsilon_3^*\frac{\partial\varphi_2}{\partial\eta}\right]}{\left[D(\varepsilon_3^* - \varepsilon_2^*) - \left(\frac{\partial\varphi_2}{\partial\xi} - \frac{\partial\varphi_3}{\partial\xi}\right) - \varepsilon_2^*\frac{\partial\varphi_3}{\partial\xi} + \varepsilon_3^*\frac{\partial\varphi_2}{\partial\xi}\right]}. \tag{1.8}$$

In addition, in region 3

$$p_3(\xi, \eta) = p_3{}^* + E(\varepsilon_3 - \varepsilon_3{}^*) = -\rho_0 D\partial\varphi_3/\partial\xi. \tag{1.9}$$

It is noted that $\tan\beta_0$ is determined from conditions (1.3) and (1.4) taking account of the ratio in the fronts of elastic longitudinal and transverse waves with $\xi = \xi_a$, $\eta = h$ (see Fig. 1). By using (1.1) and (1.2), from (1.4) and (1.6) we obtain

$$f_3'(z) = -\frac{1}{A(\lambda, G)} f_4(z + 2\mu h), \tag{1.10}$$

$$F_3''(z) = \frac{\mu(\mu_2^2 - 1)}{D\mu_1(\mu_2^2 + 1)} \{f_3'[z + h(\mu_1 - \mu)] - f_4'[z + h(\mu_1 + \mu)]\};$$

$$F_4''(z) = -\frac{2\mu}{D(\mu_2^2 + 1)} \{f_3'[z + h(\mu_2 - \mu)] - f_4'[z + h(\mu_2 + \mu)]\}; \tag{1.11}$$

$$f_4'(z) - \lambda_0 A(\lambda, G) f_4'(\lambda_0 z + K_0) = F(z), \tag{1.12}$$

where

$$K_0 = \mu[(1 - \lambda_0)h + (1 + \lambda_0)\,\mathrm{tg}\,\beta_0\xi_a];$$

$$F(z) = -\frac{A(\lambda, G)}{(1 + \mu\,\mathrm{tg}\,\beta_0)}\left\{u_2\left[\frac{z - \mu(h - \mathrm{tg}\,\beta_0\xi_a)}{(1 + \mu\,\mathrm{tg}\,\beta_0)},\ (h + \mathrm{tg}\,\beta_0\xi_a) - \right.\right. \tag{1.13}$$

$$\left.\left. - \mathrm{tg}\,\beta_0 \frac{z - \mu(h - \mathrm{tg}\,\beta_0\xi_a)}{(1 + \mu\,\mathrm{tg}\,\beta_0)}\right] - \mathrm{tg}\,\beta_0 v_2\left[\frac{z - \mu(h - \mathrm{tg}\,\beta_0\xi_a)}{(1 + \mu\,\mathrm{tg}\,\beta_0)},\ (h + \mathrm{tg}\,\beta_0\xi_a) - \mathrm{tg}\,\beta_0\frac{z - \mu(h - \mathrm{tg}\,\beta_0\xi_a)}{(1 + \mu\,\mathrm{tg}\,\beta_0)}\right]\right\},$$

$$A(\lambda, G) = \left\{1 - \frac{\mu}{\mu_1(\mu_2^2 + 1)}\left[\frac{\lambda}{\rho_0 D^2}(\mu_1^2 + 1)(\mu_2^2 - 1) + \frac{2G}{\rho_0 D^2}(\mu_1^2(\mu_2^2 - 1) + \right.\right.$$

$$\left.\left. + 2\mu_1\mu_2)\right]\right\} \Big/ \left\{1 + \frac{\mu}{\mu_1(\mu_2^2 + 1)}\left[\frac{\lambda}{\rho_0 D^2}(\mu_1^2 + 1)(\mu_2^2 - 1) + \frac{2G}{\rho_0 D^2}(\mu_1^2(\mu_2^2 - 1) + 2\mu_1\mu_2)\right]\right\}.$$

By solving functional Eq. (1.12) by the method of successive approximation it is possible to find a recurrent equation

$$f_4'(z) = F(z) + \sum_{n=1}^{\infty} \lambda_0^n A^n(\lambda, G) F\left[\lambda_0^n z + K_0 \frac{\lambda_0^n - 1}{\lambda_0 - 1}\right]. \tag{1.14}$$

It is noted that from Eq. (1.14) with $\lambda \to \infty$, $G \to \infty$, i.e., with $A(\lambda, G) = 1$, results in [2] are obtained.

The study showed that series (1.14) with $\lambda_0 < 1$ and $A(\lambda, G) < 1$ converges (in carrying out calculations it is easy to establish the radius of its convergence). Thus, from (1.1) and (1.2) taking account of (1.10), (1.11) and (1.14) the velocity field $u_3(\xi, \eta) = \partial\varphi_3/\partial\xi$, $v_3(\xi, \eta) = \partial\varphi_3/\partial\eta$ is determined for a nonlinearly compressible material in region 3 and the elastic base in regions I and II. Equations (1.7) and (1.8) make it possible to determine p_3^* and $\tan\beta(\xi)$. By using (1.9) and (1.5) we find the pressure field in the strip and stress components in the half-space, in particular at the contact line between the strip and the deforming base. Consequently, the problem in regions 3, I, and II is entirely resolved.

2. In the case when the base of the strip consists of a more compliant plastic material modelled by an ideal inelastic medium, i.e., with $\rho_p < \rho_0$ (ρ_p is density of the base material) the reflected shock wave considered in part 1 degenerates into a strong separation elastic wave and its front Σ_0 conforms with the front of the reflected elastic wave AB, and disturbed region 2 disappears (see Fig. 1). The structure of waves in a plastic half-space depends on physicomathematical characteristics and deformation laws for its materials.

We assume that deformation of the material of the base of the strip occurs by a Prandtl scheme with Young's moduli E_1 and E_2 ($E_1 > E_2$). Then after reaction of an oblique compression wave Σ with the boundary of the plastic base at first an elastic wave $a_0 = \sqrt{E_1/\rho_p}$, and then a plastic wave $a_p = \sqrt{E_2/\rho_p}$ with angles of slope γ_0 and γ will propagate, and for the base the wave picture presented in Fig. 1 is qualitatively retained. As was said previously, in this case solution of the problem in region I in [2]. In order to solve the problem in regions 3 and II we use equations [2, 4]

$$\mu^2 \frac{\partial^2 \varphi_3}{\partial \xi^2} - \frac{\partial^2 \varphi_3}{\partial \eta^2} = 0, \quad \mu^2 = \left(\frac{D}{c_p}\right)^2 - 1; \tag{2.1}$$

$$\nu_0^2 \frac{\partial^2 \varphi_{II}}{\partial \xi^2} - \frac{\partial^2 \varphi_{II}}{\partial \eta^2} = 0, \quad \nu_0^2 = \left(\frac{D}{a_0}\right)^2 - 1. \tag{2.2}$$

Equations (2.1) and (2.2) permit solutions in the form

$$\begin{aligned} \varphi_3(\xi, \eta) &= \Phi_1(\xi - \mu\eta) + \Phi_2(\xi + \mu\eta), \\ \varphi_{II}(\xi, \eta) &= \Phi_3(\xi - \nu_0\eta) + \Phi_4(\xi + \nu_0\eta). \end{aligned} \tag{2.3}$$

In order to find unknown functions Φ_i ($i = \overline{1, 4}$) the problem in question has the following boundary conditions:

$$(v_3 - v_1) = \mu(u_3 - u_1) \quad \text{for} \quad \eta = h - (\xi - \xi_a)/\mu; \tag{2.4}$$

$$p_3(\xi, \eta) = p_{II}(\xi, \eta), \; v_2(\xi, \eta) = v_{II}(\xi, \eta) \quad \text{for} \quad \eta = h, \; \xi_a \leqslant \xi \leqslant \xi_c; \tag{2.5}$$

$$\operatorname{tg} \gamma(v_{II} - v_I) = -(u_{II} - u_I) \quad \text{for} \quad \eta = h + \operatorname{tg} \gamma(\xi - \xi_a) \tag{2.6}$$

($a_p = D \sin\gamma$, h is strip thickness). Considering that $p_3(\xi, \eta) = -\rho_0 D u_3(\xi, \eta)$, $p_{II}(\xi, \eta) = -\rho_p D u_{II}(\xi, \eta)$, we substitute (2.3) in (2.4)-(2.6). Then after some transformations we obtain a functional equation

$$\Phi_4'(z) + \lambda_p \Phi_4'(\lambda_1 z + K_1) = G(z), \tag{2.7}$$

where

$$G(z) = \frac{(\operatorname{tg} \gamma v_I + u_I)}{(1 + \nu_p \operatorname{tg} \gamma)} - \frac{\lambda_1}{(\nu_0/\mu + \rho \;\; /\rho_0)} \left\{ u_1[\psi_1(z), -\psi_2(z)] - \frac{1}{\mu} v_1[\psi_1(z), -\psi_2(z)] \right\};$$

$$\psi_1(z) = \frac{\lambda_1 [z - \nu_0(h - \operatorname{tg} \gamma \xi_a)] + (1 + \nu_0 \operatorname{tg} \gamma) \xi_a}{2};$$

$$\psi_2(z) = \frac{\lambda_1 [z - \nu_0(h - \operatorname{tg} \gamma \xi_a)] - (1 - \nu_0 \operatorname{tg} \gamma) \xi_a - 2\mu h}{2\mu};$$

$$\lambda_1 = \frac{(1 - \nu_0 \operatorname{tg} \gamma)}{(1 + \nu_0 \operatorname{tg} \gamma)}; \quad m = \left(\frac{\nu_0}{\mu} - \frac{\rho_p}{\rho_0}\right) \Big/ \left(\frac{\nu_0}{\mu} + \frac{\rho_p}{\rho_0}\right);$$

$$\lambda_p = \lambda_1 m; \quad k_1 = \nu_0[(h + \operatorname{tg} \gamma \xi_a) - \lambda_1(h - \operatorname{tg} \gamma \xi_a)].$$

As in [2], Eq. (2.7) is solved by the method of successive approximations. Then

$$\Phi_4'(z) = G(z) + \sum_{n=1}^{\infty} (-\lambda_p)^n G\left[\lambda_1^n z + K_1 \frac{(\lambda_1^n - 1)}{(\lambda_1 - 1)}\right]. \tag{2.8}$$

Since $\lambda_p < 1$, it is possible to demonstrate convergence of series (2.8) which is confirmed by numerical calculations.

Thus, in order to determine velocity and pressure components in regions 3 and II of a layered plastic medium we have

$$\begin{aligned} u_3(\xi, \eta) = {} & \frac{1}{2}[u_1(\chi_1, \chi_2) - v_1(\chi_1, \chi_2)/\mu] + \frac{2\rho_p \nu_0/(\rho_0 \mu)}{(\nu_0/\mu + \rho_p/\rho_0)} \Big\{ G[(\xi + \mu\eta) + \\ & + (\nu_0 - \mu)h] + \sum_{n=1}^{\infty} (-\lambda_p)^n G[\lambda_1^n(\xi + \mu\eta + (\nu_0 - \mu)h) + K_1(\lambda_1^n - 1)/(\lambda_1 - 1)] \Big\} - \\ & - \frac{m}{2}[u_1(\chi_3, \chi_4) - v_1(\chi_3, \chi_4)/\mu], \end{aligned}$$

$$\begin{aligned} v_3(\xi, \eta) = {} & -\frac{\mu}{2}[u_1(\chi_1, \chi_2) - v_1(\chi_1, \chi_2)/\mu] + \frac{2\rho_p \nu_0/\rho_0}{(\nu_0/\mu + \rho_p/\rho_0)} \Big\{ G[(\xi + \mu\eta) + \\ & + (\nu_0 - \mu)h] + \sum_{n=1}^{\infty} (-\lambda_p)^n G[\lambda_1^n(\xi + \mu\eta + (\nu_0 - \mu)h) + \\ & + K_1(\lambda_1^n - 1)/(\lambda_1 - 1)] \Big\} - \frac{m\mu}{2}[u_1(\chi_3, \chi_4) - v_1(\chi_3, \chi_4)/\mu], \; p_3(\xi, \eta) = -\rho_0 D u_3(\xi, \eta); \end{aligned}$$

$$u_{II}(\xi,\eta) = m\Big\{G[(\xi - v_0\eta) + 2v_0 h] + \sum_{n=1}^{\infty}(-\lambda_p)^n G\left[\lambda_1^n(\xi - v_0\eta + 2v_0 h) + K_1(\lambda_1^n - 1)/(\lambda_1^n - 1)\right]\Big\} + \frac{1}{(v_0/\mu + \rho_p/\rho_0)}[u_1(v_1, v_2) - v_1(v_1, v_2)/\mu] + G(\xi + v_0\eta) + \sum_{n=1}^{\infty}(-\lambda_p)^n G\left[\lambda_1^n(\xi + v_0\eta) + K_1(\lambda_1^n - 1)/(\lambda_1 - 1)\right],$$

$$v_{II}(\xi,\eta) = -m\mu\Big\{G[(\xi - v_0\eta) + 2v_0 h] + \sum_{n=1}^{\infty}(-\lambda_p)^n G\left[\lambda_1^n(\xi - v_0\eta + 2v_0 h) + K_1(\lambda_1^n - 1)/(\lambda_1 - 1)\right]\Big\} - \frac{\mu}{(v_0/\mu + \rho_p/\rho_0)}[u_1(v_1, v_2) - v_1(v_1, v_2)/\mu] + \mu\Big\{G(\xi + v_0\eta) + \sum_{n=1}^{\infty}(-\lambda_p)^n G\left[\lambda_1^n(\xi + v_0\eta) + K_1(\lambda_1^n - 1)/(\lambda_1 - 1)\right]\Big\}, \quad (2.9)$$

$$p_{II}(\xi,\eta) = -\rho\, D u_{II}(\xi,\eta),$$

where

$$\chi_1 = \frac{(\xi - \mu\eta) + \mu(h + \xi_a/\mu)}{2};\quad \chi_2 = \frac{(h + \xi_a/\mu) - (\xi - \mu\eta)/\mu}{2};$$

$$\chi_3 = \frac{(\xi + \mu\eta) - \mu(h - \xi_a/\mu)}{2};\quad \chi_4 = \frac{(h + \xi_a/\mu) - (\xi + \mu\eta - 2\mu h)/\mu}{2};$$

$$v_1 = \frac{(\xi - v_0\eta) + v_0 h + \xi_a}{2};\quad v_2 = -\frac{(\xi - v_0\eta) + v_0 h - \mu(2h + \xi_a/\mu)}{2\mu}.$$

Furthermore, parameters of the plastic compliant base of the strip in region I (see Fig. 1) with fulfillment of the condition $p_I(\xi, \eta) = \sigma_s$ (σ_s is a prescribed constant value) are determined as:

$$u_I = -\sigma_s/\rho_p D,\ v_I = \sigma_s/\rho_p D\ \mathrm{tg}\,\gamma_0,\ \varepsilon_I = \sigma_s/E_1\,(\mathrm{tg}\,\gamma_0 = 1/v_0).$$

In this analytical study the problem of reaction of a moving load on a two-layer plastic half-space is assumed to be complete. However, it is emphasized that this procedure makes it possible to study shock-wave processes in a layered half-space of different structure and in the case of a shock wave in the material of the plastic base of the strip.

3. On the basis of the equations obtained above calculations were carried out on a computer for parameters of movement and pressure, including stresses, for a two-layer material consisting of a layer of soft ground of various construction [2, 4] and a deformable base. Marble and epoxy foam were taken as the base materials. In order to calculate their parameters the following data and relationships were used [2]:

$$\eta(\xi) = (\mathrm{tg}\,\alpha_0 - (b/2)\xi)\xi,\ \eta'(\xi) = d\eta/d\xi > 0;$$

$$\rho_0 = 200\ \mathrm{kg\cdot sec^2/m^4}\quad \alpha_1 = 12.127\cdot 10^2\ \mathrm{kg/cm^2},\quad \alpha_2 = 58.73\cdot 10^3\ \mathrm{kg/cm^2}, \quad (3.1)$$

$$E = 14\cdot 10^3\ \mathrm{kg/cm^2}\quad h = 1.5\ \mathrm{m},\ b = 0.86\cdot 10^{-3}(1/\mathrm{m}),$$

$$p_0 = f(0) = 300\ \mathrm{kg/cm^2};$$

$$\rho_{ba} = 260\ \mathrm{kg\cdot sec^2/m^4};\ a_0 = 4000\ \mathrm{m/sec}\quad b_0 = 2200\ \mathrm{m/sec}; \quad (3.2)$$

$$\rho_0 = 200\ \mathrm{kg\cdot sec^2/m^4},\ \alpha_1 = 18\cdot 10^2\ \mathrm{kg/cm^2},\ \alpha_2 = 82\cdot 10^3\ \mathrm{kg/cm^2}, \quad (3.3)$$

$$E = 18\cdot 10^3\ \mathrm{kg/cm^2},\quad h = 3.5\ \mathrm{m},\ b = R_2 = 0.2301\cdot 10^{-3}\ (1/\mathrm{m}),$$

$$p_0 = 100\ \mathrm{kg/cm^2},\quad D = 3114\ \mathrm{m/sec},\ \mathrm{tg}\,\alpha_0 = R_1 = 0.1433;$$

$$\rho_p = 20\ \mathrm{kg\cdot sec^2/m^4},\ E_1 = 250\ \mathrm{kg/cm^2},\ E_2 = 150\ \mathrm{kg/cm^2},\quad \sigma_s = 3.5\ \mathrm{kg/cm^2},\ \mathrm{tg}\,\beta = 1/\mu,\ \mu^2 = (D^2/c_p^2 - 1). \quad (3.4)$$

Some results of calculations in the form of curves for the change in pressure p, vertical v and horizontal u components of velocity of the medium in relation to horizontal coordinate ξ taking account of (3.1)-(3.4) are given in Figs. 2-5 where curves 1-3 correspond to horizontal levels η = 0; 3h/4; in Figs. 4 and 5 h relates to solution of the problem in part 2 using original data from (3.3) and (3.4) (curve 2 is η = h/2).

It can be seem from Figs. 2-5 that p and v in region 1 with an increase in h decrease steadily. The reduction in p and v over the vertical coordinate η depends markedly on the nonlinear properties of the ground and the loading profile. In the ground the wave process

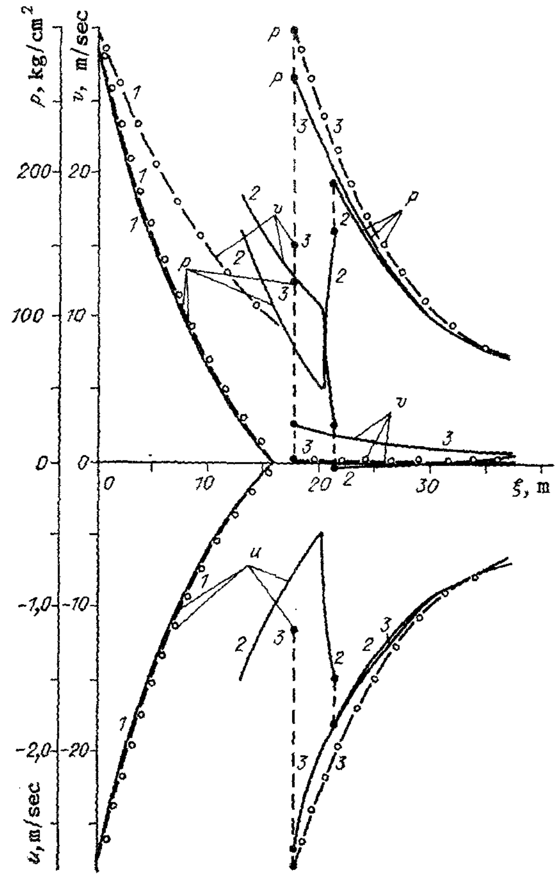
p, kg/cm²
v, m/sec
u, m/sec
ξ, m

Fig. 2

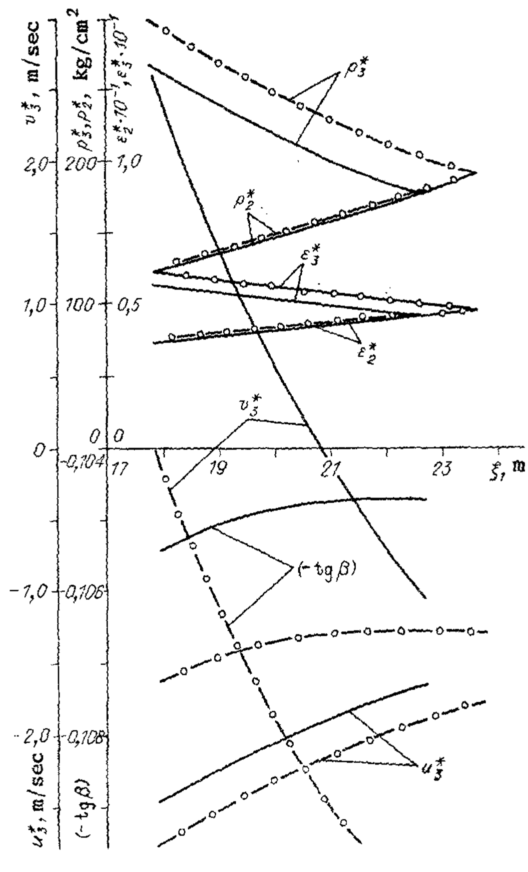
v_3^*, m/sec
p_3^*, p_2^*, kg/cm²
u_3^*, m/sec
(− tg β)
ξ_1, m
p_3^*
p_2^*
ε_3^*
ε_2^*
v_3^*
u_3^*

Fig. 3

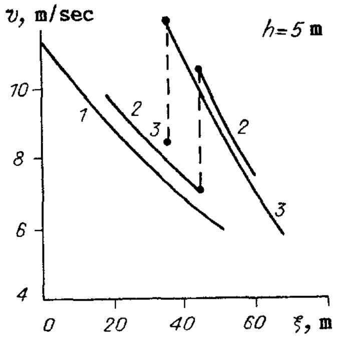

Fig. 4

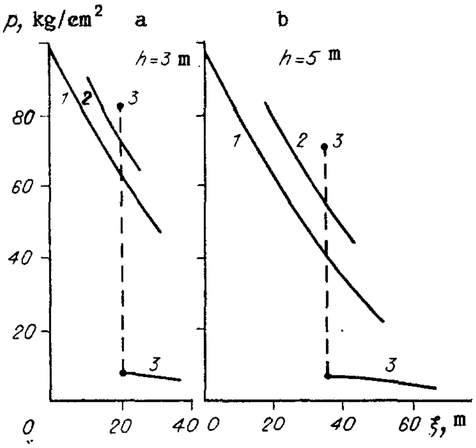

Fig. 5

transformed as follows: ground with different horizontal levels at first under the action of a compression wave is loaded instantaneously to a certain stress level, and behind the front of this wave in region 1 with respect to ξ there is unloading and pressure falls. Then in region 2 with reaction of an oblique compression wave with the rocky base of the strip there is some gradual increase in pressure. In this case at the instant of action of a reflection of the waves from the contact of the two media a further jumpwise increase (reduction) of p(v) is obtained followed by fading of their amplitude in region 3 with respect to ξ (Figs. 2 and 3). However, if the base of the strip serves as a plastically compliant low modulus material (epoxy foam, polystyrene foam, etc.), then as calculations show the reflected wave (Figs. 4 and 5) becomes an unloading wave and at the front of this wave in contrast to the case of a rocky base there is a jumpwise reduction (increase) in the value of pressure (vertical component of velocity for the medium). This is due to the fact that for a layer of soft ground provided with a base of porous or lighter ground the compression wave propagating in the ground during reaction with interfaces of the media reflects from it not a shock wave, but an elastic wave with known constant velocity c_p. The front of this wave is a separation surface, and consequently in it with $\eta = h$ a jump in pressure arises. A similar picture is observed also with reflection of an elastic or acoustic wave from a free surface [5].

By analyzing the curves in Fig. 3 it was detected that pressure p_3^* (p_2^*) from the direction of region 3 (2) along the front reflected from the elastic and more dense ground of the base depending on ξ the wave gradually falls (increases) and the vertical (horizontal) component of velocity for the medium increases (decreases). The curve for p_3^* (p_2^*) exhibits some lower (increased) amplitude than the corresponding curve from [2] for strips lying on a rigid base (see Fig. 2, broken lines with circles). Pressure distribution in the reflected wave front and in region 3 depends markedly on its profile which exists at the upper boundary of the strip with the moving load. For example, in the case of $b = 0.86 \cdot 10^{-3}$ the reflected wave at instant $\xi = 22.6$ m is extinguished, i.e., pressure p_3^* becomes equal to p_2^* (see Fig. 3, solid lines). If we ignore the deformability of the base, then the duration of action of the reflected wave on the medium is somewhat prolonged.

By studying curves $\tan\beta(\xi)$ (Fig. 3) it is noted that with an increase in ξ it decreases slowly, and consequently the reflected wave front becomes curved surface towards axis $O\xi$. In addition, curve $\tan\beta(\xi)$ taking account of deformability of the base is located beneath the curve obtained for a wave reflected from the rigid base of the strip (broken line with circles).

On the whole the study and comparative analysis of calculated results show that in considering the problem with a rigid or deformable base more dense than the ground the maximum value of contact pressure behind the reflected wave front exceeds the value of pressure at the corresponding point of the descending wave by more than a factor of two, and with presence of a plastic compliant base the level of pressure at the contact line of the media decreases somewhat, i.e., in the last case the reflected wave is a strong distortional elastic wave, or at its surface there is a jumpwise reduction in the value of pressure.

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AXISYMMETRICAL CONTACT PROBLEMS FOR PRESTRESSED DEFORMABLE BODIES

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Axisymmetrical contact problems are considered for a half-space and a layer of finite thickness h lying without friction on a deformable base previously stressed by uniform forces with intensity p applied at infinity. It is assumed that the material of deformable bodies is described by equations of physical nonlinear elasticity theory. The initial stress-strained state of the bodies (prestressing regime) is determined as an accurate solution of these equations. Action of a load on the surface of a layer (half-space) is considered as a small disturbance of the basic nonlinear stress field caused by prior loading. This makes it possible to perform linearization of all equations with respect to additional stresses, strains, and displacements. Contact problems for impression of a rigid stamp into a physically nonlinear material are posed for the linear equations obtained which are then reduced to first-order integral equations with a symmetrical irregular kernel with respect to distribution functions for contact pressures. Solutions of these equations are built up by means of asymptotic methods. Cases of loss of stability and deformability of a medium as a result of prestressing are studied. The effect of prestressing regime on the magnitude of contact pressures is studied.

1. Resolution equations for physically nonlinear (geometrically linear) elasticity theory for the case of axial symmetry and with the condition of absence of mass forces may be written as follows [1]:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\varphi}{r} = 0, \ \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_r}{\partial z} + \frac{\tau_{rz}}{r} = 0; \tag{1.1}$$

$$\varepsilon_r = \psi\sigma_r + (\varphi - \psi)\sigma, \ \varepsilon_\varphi = \psi\sigma_\varphi + (\varphi - \psi)\sigma, \tag{1.2}$$

$$\varepsilon_z = \psi\sigma_z + (\varphi - \psi)\sigma, \quad \varepsilon_{rz} = \psi\tau_{rz}, \quad \varepsilon = \varphi\sigma, \quad \gamma = 2\psi\tau,$$

$$\sigma = (\sigma_r + \sigma_\varphi + \sigma_z)/3, \quad \varepsilon = (\varepsilon_\varphi + \varepsilon_r + \varepsilon_z)/3,$$

$$\tau = \frac{1}{\sqrt{6}}\left[(\sigma_r - \sigma_\varphi)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_\varphi - \sigma_z)^2 + 6\tau_{rz}^2\right]^{1/2},$$

$$\gamma = \sqrt{\frac{2}{3}}\left[(\varepsilon_r - \varepsilon_\varphi)^2 + (\varepsilon_r - \varepsilon_z)^2 + (\varepsilon_\varphi - \varepsilon_z)^2 + \frac{3}{2}\left(\gamma_{r\varphi}^2 + \gamma_{rz}^2 + \gamma_{\varphi z}^2\right)\right]^{1/2},$$

$$\varepsilon_r = \partial u/\partial r, \ \varepsilon_\varphi = u/r, \ \varepsilon_z = \partial w/\partial z,$$

$$\varepsilon_{rz} = (1/2)(\partial u/\partial z + \partial w/\partial r), \ u = u(r, z), \ w = w(r, z),$$

$$\frac{\partial^2 \varepsilon_r}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial r^2} = 2\frac{\partial^2 \varepsilon_{rz}}{\partial r\, \partial z}, \ \frac{\partial \varepsilon_\varphi}{\partial r} = \frac{\varepsilon_r - \varepsilon_\varphi}{r}.$$

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